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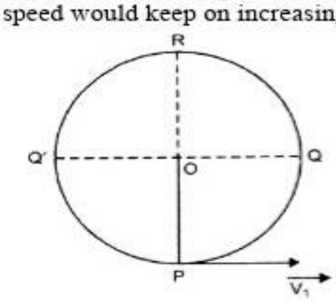
The motion of a particle in the vertical circle is different to the

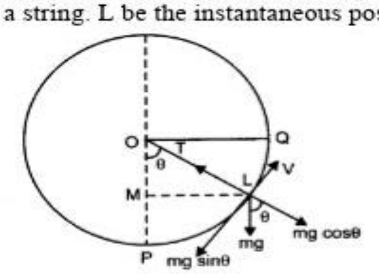
Motion of a particle in horizontal circle. In

horizontal circle, the motion is not effected by the acceleration due to gravity (g) whereas in the motion of vertical circle, the motion is not effected by the acceleration due to gravity (g) whereas in the motion of vertical circle, the value of 'g' plays an important role, the motion in this case does not remain uniform. When the particle move up from its lowest position P, its speed continuously decreases till it reaches the highest point of its circular path. This is due to the work done against the force of gravity. When the particle moves down the circle, its speed would keep on increasing.



(i)

Let us consider a particle moving in a circular vertical path of radius V and centre o tide with a string. L be the instantaneous position of the particle such that



(ii)

$\angle POL = \theta°$

Here the following forces act on the particle of mass 'm'.

(i) Its weight = mg (verticaly downwards).

(ii) The tension in the string T along LO.

The instantaneous velocity of the particle at L is $\vec{V}$ along the direction of the tangent to the circle.

Centripetal force on the particle $= \dfrac{mV^2}{r}$

$$\therefore \quad \frac{mV^2}{r} = T - mg\cos\theta \qquad \text{[From Fig. (ii)]}$$

$$\text{So} \quad T = \frac{mV^2}{r} + mg\cos\theta \qquad (i)$$

We can take the horizontal direction at the lowest point 'p' as the position of zero gravitational potential energy. Now as per the principle of conservation of energy, The total energy at P =Total energy at L

$$\frac{1}{2}mV_1^2 + 0 = \frac{1}{2}mV^2 + mgh$$

$$\Rightarrow \quad V_1^2 = V^2 + 2mgh \qquad (ii)$$

From right angled $\Delta\, OML$,

$$OM = OL \cos\theta$$
$$= r\cos\theta$$

$$\therefore \quad MP = h = OP - OM$$
$$= r - r\cos\theta$$
$$= r(1 - \cos\theta) \qquad (iii)$$

Substituting the value of 'h' in eq. (ii) we get,

$$V_1^2 = V^2 + 2gr(1 - \cos\theta) \qquad (iv)$$

Now substituting the value of V^2 from (iv) to (i), we get

$$T = \frac{m}{r}\left[V_1^2 - 2gr(1 - \cos\theta)\right] + mg\cos\theta$$

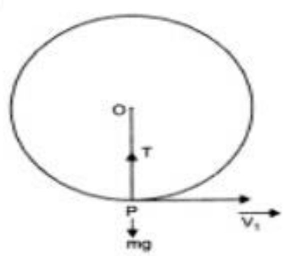
$$T = \frac{mV_1^2}{r} - 2mg(1 - \cos\theta) + mg\cos\theta$$

$$= \frac{mV_1^2}{r} - 2mg + 2mg\cos\theta + mg\cos\theta$$

$$= \frac{mV_1^2}{r} - 2mg + 3mg\cos\theta \qquad (v)$$

From this relation, we can calculate the tension in the string at the lowest point P, mid-way point and at the highest position of the moving particle.

Case (i) : At the lowest point P, $\theta = 0°$



$$T_P = \frac{mV_1^2}{r} - 2mg + 3mg\cos 0°$$

$$= \frac{mV_1^2}{r} - 2mg + 3mg$$

$$= \frac{mV_1^2}{r} + mg \qquad (vi)$$

Hence, the tension in the string at the lowest point P is,

$$T = \frac{mV_1^2}{r} + mg$$